Laser-generated nonlinear Rayleigh waves with shocks

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(Received 16 January 1999; accepted for publication 18 December 1998)

Intense laser radiation was used to generate a Rayleigh wave pulse of finite amplitude in fused quartz. Measurements of the pulse at two locations along the propagation path reveal the formation of well-defined shocks in the horizontal (in-plane) velocity waveform. As the pulse propagates, the different propagation speeds of the head and tail shocks lead to a considerable increase in the duration of the pulse. Theoretical predictions based on nonlinear spectral evolution equations are in close agreement with the observed waveform distortion and shock formation. © 1999 Acoustical Society of America. [S0001-4966(99)03704-2]

PACS numbers: 43.25.Fe [MAB]

INTRODUCTION

Rayleigh waveform distortion observed by Lomonosov and Hess1 can be interpreted as a nonlinear evolution process culminating in the formation of well-defined shocks. Both generation and detection of the surface waves were performed with laser beams. The vertical (normal with respect to the interface) component of the particle velocity waveform measured at two different locations along the propagation path of a single Rayleigh wave pulse are presented, together with calculations of the corresponding horizontal (in-plane) component and frequency spectrum. The observed waveform evolution between the two detection points is compared here with theoretical predictions obtained from spectral evolution equations derived by Zabolotskaya.2

Laser irradiation is a common noncontact method for generating elastic waves in solids.3 In nondestructive testing, the laser intensity is limited to the thermoelastic mode of operation, and the efficiency of elastic wave generation is typically so low that propagation of the resulting surface waves is adequately described by linear theory.4 At higher laser intensities, at which optical breakdown occurs, surface waves have been generated with amplitudes so large that microparticles are ejected from the surface along the propagation path.5

The efficiency of surface wave generation in the thermoelastic mode of operation can be increased considerably by introducing on the surface of the solid a thin layer of liquid having a large optical absorption coefficient. In this method, absorption of the laser radiation produces intense evaporation of the liquid coating, and the resulting reaction pressure leads to generation of the surface wave. Generation of surface waves using a liquid coating has been shown to be substantially more efficient than generation by direct laser irradiation, in the absence of liquid coupling, even when optical breakdown occurs.6 Laser-induced photoacoustic generation incorporating a liquid layer was the method used to produce the measured Rayleigh waves reported here.

Theories for nonlinear surface waves that were advanced in the 1980s7–9 were motivated in part by the development of surface acoustic wave devices for nonlinear signal processing operations. The corresponding predictions of harmonic generation and waveform distortion were restricted to the preshock region. In contrast with these earlier models, the theory employed here to interpret measurements of nonlinear Rayleigh waves was derived using Hamiltonian mechanics.2,10 Because the evolution equations follow directly from the Hamiltonian for nonlinear surface waves, the theory can be extended in a straightforward way to include effects such as anisotropy and piezoelectricity. This model has been used in theoretical investigations of waveform distortion2 and harmonic generation11 out to distances well beyond where shocks are formed.

We note that a similar comparison of theory and experiment was published recently by Kolomenskii et al.12 In their work, waveforms were compared with calculations based on an evolution equation developed by Gusev et al.13 One way their work differs from ours is that the material constants in their theory were adjusted to provide a best fit with the measurements. We do not employ curve fitting, and we use measurements of the third-order elastic constants that are published in the literature. Another difference is that the present theory describes the cusped wave profile near the shocks in the measured waveforms, a feature that has been discussed in detail previously2,11 and which distinguishes the nonlinear distortion of surface waves from that of bulk waves.

I. EXPERIMENT

Our experiments investigated the propagation of nonlinear Rayleigh waves in fused quartz. The material is consid-
pered to be isotropic, and the small-signal elastic properties are characterized by the longitudinal wave speed \( c_l = 5960 \text{ m/s} \), transverse wave speed \( c_t = 3754 \text{ m/s} \), and density \( \rho = 2203 \text{ kg/m}^3 \), for which the corresponding Rayleigh wave speed is \( c_R = 3401 \text{ m/s} \). Rayleigh wave pulses were generated by shining Nd:YAG laser radiation onto the material. The pulsed laser radiation was focused with a cylindrical lens to form a strip having dimensions 6 mm by 50 \( \mu \text{m} \) on the surface of the solid (see Fig. 1). The wavelength of the laser radiation was 1064 nm, the pulse length 7 ns, and optical pulse energies up to 100 mJ were used. At the location where the sample was irradiated by the laser, the surface of the solid was covered by a liquid layer having a large optical absorption coefficient.

The Rayleigh wave pulse generated by laser excitation was detected at two locations on the surface of the solid. The two locations were at distances 2.3 mm and 18.3 mm along a direction normal to the long (6 mm) dimension of the irradiated strip. A laser beam deflection technique was used to make the measurement at each location. A second Nd:YAG laser was used for this purpose, and the probe beams were formed by the second harmonic component of the cw laser radiation (532 nm, 30 mW). The spot size of each probe beam on the surface was 4 \( \mu \text{m} \). Deflection of each beam was detected by a separate fast photodiode, each possessing 500-MHz bandwidth. The detected signal is proportional to the slope of the surface, and therefore, for a traveling surface wave, proportional to the vertical component of the surface velocity.

II. RESULTS

A vertical velocity waveform \( v_z \) measured at the first location, 2.3 mm from the source, is shown in Fig. 2(a). The corresponding horizontal velocity waveform \( v_x \) in Fig. 2(b) was calculated by taking the Hilbert transform of the vertical component:

\[
v_x(t) = -\left( \frac{\eta + \xi_t}{1 + \eta \xi_t} \right) \frac{1}{\pi} \text{Pr} \int_{-\infty}^{\infty} \frac{v_z(t')}{t'-t} dt'.
\]

where \( \xi_t = (1 - c_T^2 / c_l^2)^{1/2} \), \( \xi_t = (1 - c_T^2 / c_l^2)^{1/2} \), and \( \eta = -2 \xi_t / (1 + \xi_t^2) \). We have taken \( x \) to be the coordinate along the direction of propagation, and \( z \) to be the coordinate...
normal to, and defined positive outward from, the surface of the solid. The frequency spectrum of the signal, normalized by its peak value, is shown in Fig. 2(c).

We first assess the potential influence of diffraction on the propagation of the pulse. The characteristic diffraction length for a Rayleigh wave with angular frequency $\omega$ radiated from a line source with half-width $a$ is $x_d = k a^2 / 2$, where $k = \omega c / c_R$. Taking $a = 3$ mm and, from Fig. 2(c), $\omega / 2 \pi = 20$ MHz to characterize the dominant frequency band, yields $x_d = 166$ mm, which is an order of magnitude greater than the distance where the second measurement was made along the propagation path, $x = 18.3$ mm. Although the effect of diffraction was included in an earlier theoretical investigation of nonlinear Rayleigh wave beams,14 its effect here is negligible and it is sufficient to use plane wave theory to interpret the observed waveform distortion.

The horizontal velocity waveform in Fig. 2(b) may be used to estimate the shock formation distance and thus the effect of nonlinearity over the propagation distance $\Delta x = 16$ mm between the two observation points. As discussed below, it is nonlinear distortion of the horizontal component, rather than the vertical component, that resembles the distortion of finite amplitude sound in fluids. The plane wave shock formation distance $x_R$ for an initially sinusoidal Rayleigh wave in a lossless medium may be expressed in the same form as for sound waves in fluids:

$$x_R = \frac{1}{|\beta| \epsilon_x k}, \quad (2)$$

where $\beta$ is the coefficient of nonlinearity, and $\epsilon_x = v_{x0}/c_R$ is the peak acoustic Mach number (or equivalently, strain) associated with the initial waveform $v_{x}(x,t) = v_{x0} \sin \omega t$. An approximate analytical expression for the coefficient of nonlinearity is given by Eq. (32) in Ref. 15. The value $\beta = -2.3$ is thus obtained for fused quartz from the values of the two second-order elastic constants, the bulk modulus $K = 36.9$ GPa and shear modulus $\mu = 31.0$ GPa (corresponding to the values cited above for $c_1$, $c_3$, and $\rho$), and the three third-order elastic constants measured by Bogardus.16 Expressed in the notation of Landau and Lifshitz,17 these third-order constants are $A = -42$ GPa, $B = 93$ GPa, and $C = 26$ GPa. Letting $v_{x0} = 30$ m/s and $\omega / 2 \pi = 20$ MHz characterize the waveform in Fig. 2(b), one obtains $x_R = 1.3$ mm from Eq. (2).

The predicted shock formation distance is thus an order of magnitude less than the propagation distance $\Delta x = 16$ mm between the two locations where the waveforms were detected, which is consistent with the strong nonlinear distortion observed at the second location, shown as solid lines in Fig. 2(d) and (e). The negative value obtained for $\beta$ indicates that nonlinearity causes positive portions of the horizontal velocity waveform to propagate with local phase speeds that are slower than the small-signal speed $c_R$ of zero crossings, and negative portions to propagate faster, also in agreement with the distortion process observed by comparing Fig. 2(b) and (e). Because the negative head shock in the horizontal velocity waveform propagates faster than the positive tail shock, the pulse duration approximately doubles between the two observation points, which accounts for the corresponding spectral shift observed by comparing Fig. 2(c) and (f). Such distortion is opposite that of sound waves in fluids in the sense that positive portions of the acoustical particle velocity steepen forward and thus advance in time, whereas negative portions recede. However, the value of $\beta$ may be either positive or negative for Rayleigh waves, and for most of the materials considered previously15 it is positive.

The theoretical predictions shown as dashed lines in Fig. 2(d)-(f) were obtained by assuming plane wave propagation between the two observation points and solving the following set of coupled spectral equations2 numerically:

$$\frac{dv_n}{dz} + \alpha_n v_n = -\frac{\mu \omega_n^2}{2 \rho c^4 R} \sum_{l+m=n} \frac{m}{|m|} R_{lm} v_l v_m, \quad (3)$$

where $v_n(x)$ is the amplitude of the $n$th harmonic component, $\omega_n$ the angular repetition frequency for periodic signals (corresponding to the fundamental component $n = 1$), $\xi$ a combination of second-order elastic constants, and $R_{lm}$ a nonlinearity matrix that depends on both second- and third-order elastic constants.11 The absorption terms $\alpha_n v_n$ were introduced ad hoc to ensure numerical stability when shocks develop in the waveforms. The horizontal and vertical velocity waveforms at the surface of the solid are constructed from the complex spectral amplitudes as follows:

$$v_{x}(x,t) = (\eta + \xi t) i \sum_{n=-\infty}^{\infty} \frac{v_n(x) e^{-i \omega_n (t - x/c_R)}}{|n|}, \quad (4)$$

$$v_{z}(x,t) = (1 + \xi t) i \sum_{n=-\infty}^{\infty} \frac{v_n(x) e^{-i \omega_n (t - x/c_R)}}{|n|}. \quad (5)$$

Equations (4) and (5) are related by Eq. (1). Equations (3) were integrated with a fourth-order Runge-Kutta routine to simulate propagation of the signal measured at $x = 2.3$ mm out to the second observation point, $x = 18.3$ mm.18 The pulse at $x = 2.3$ mm was assumed to repeat periodically every $T_0 = 0.255$ $\mu$s, and therefore the fundamental frequency in the Fourier series expansion is $\omega_0 / 2 \pi = 3.92$ MHz. The computations were performed with $N = 800$ harmonics (i.e., $1 \leq n \leq N$, with $v_{-n} = v_{n}^*$. Initial values for $v_n$ were determined via spectral decomposition of the measured signal shown in Fig. 2(a).

The absorption coefficients were chosen by assuming classical energy dissipation due to viscosity and heat conduction, for which the quadratic frequency dependence $\alpha_n = n^2 \alpha_1$ is obtained.19 Numerical values were assigned to the absorption coefficients by selecting $1 / \alpha_n = 500 x _R$, where $m$ designates the value of $n$ for the harmonic component having a frequency characteristic of the initial waveform (here $m = 6$, corresponding to $\omega / 2 \pi = 23.5$ MHz). With the absorption length chosen to be substantially larger than both the shock formation distance and the total propagation distance, the effect of absorption is assumed to be very weak in comparison with that of nonlinearity, and its main influence is on the rise times of the shocks. All remaining coefficients, the
parameter $\zeta$ and matrix elements $R_{lm}$, were calculated directly from values of the fundamental material constants $(\rho, \mu, K, A, B, C)$.

Comparison of the dashed and solid lines in Fig. 2(d) and (e) reveals close agreement between theory and experiment. Consider first the horizontal velocity waveforms in Fig. 2(e). The appearance of cusped spikes at the shocks, which was predicted to be a distinctive characteristic of nonlinear Rayleigh waves, is clearly manifest in the measured waveform, and the calculations accurately reproduce this feature. The cusping at the shocks in the horizontal velocity waveform is associated with nonlocal nonlinear effects, which are absent from the exclusively local nonlinear distortion process for sound waves in fluids. Nonlinearity causes sound waves to develop sawtoothlike profiles devoid of spikes. Measured and predicted vertical velocity waveforms are compared in Fig. 2(d), which exhibit impulses where shocks appear in the horizontal velocity waveforms. Discrepancies between the measured and predicted amplitudes of the spikes are due to experimental and numerical limitations.

Calculations were also performed with the third-order constants measured by Yost and Breazeale for fused quartz. Their values yield $\beta_4 = -2.1$, and therefore waveform distortion is predicted to occur approximately 10% more slowly than on the basis of the constants measured by Bogardus and used in the calculations for Fig. 2. Apart from this difference in nonlinear length scales, the waveforms calculated with both sets of constants are virtually identical.

III. SUMMARY

Close quantitative agreement with measurements of shock formation in nonlinear Rayleigh waves was achieved with predictions based on Zabolotskaya’s theoretical model. Not only does the theory predict the rate of nonlinear evolution, but also the fine structure associated with cusping of the waveform near the shocks, a feature that distinguishes nonlinear distortion of surface waves from that of bulk compressional and shear waves. No curve fitting was employed, with all necessary third-order elastic constants taken from the literature.

ACKNOWLEDGMENTS

Yu. A. Il’inskii is thanked for helpful discussions of this work. The experiments were performed while one of the authors (A.L.) was on leave at University of Heidelberg with financial support from Volkswagen-Stiftung, the International Science Foundation, and the Russian Foundation for Basic Research. The authors on the U.S. side were supported by the National Science Foundation and the Office of Naval Research.